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A HYPERSONIC VEHICLE MODEL DEVELOPED WITH PISTON THEORY (PREPRINT)

Michael W. Oppenheimer
David B. Doman

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/s/

Michael W. Oppenheimer
Electronics Engineer
Control Design and Analysis Branch
Air Force Research Laboratory
Air Vehicles Directorate

/s/

David B. Doman
Acting Chief
Control Design and Analysis Branch
Air Force Research Laboratory
Air Vehicles Directorate

/s/

Brian W. Van Vliet
Chief
Control Sciences Division
Air Force Research Laboratory
Air Vehicles Directorate

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14. ABSTRACT

For high Mach number flows, $M \geq 4$, piston theory has been used to calculate the pressures on the surfaces of a vehicle. In a two-dimensional flow, a perpendicular column of fluid stays intact as it passes over a solid surface. Thus, the pressure at the surface can be calculated assuming the surface were a piston moving into a column of fluid. In this work, piston theory is used to calculate the rigid body forces, moments, and stability derivatives of a hypothetical hypersonic vehicle. Only longitudinal motion is considered in this case and lateral motion will be included in subsequent work.

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A Hypersonic Vehicle Model Developed With Piston Theory

Michael W. Oppenheimer *

David B. Doman †

Air Force Research Laboratory, WPAFB, OH 45433-7531

Abstract

For high Mach number flows, $M \geq 4$, piston theory has been used to calculate the pressures on the surfaces of a vehicle. In a two-dimensional flow, a perpendicular column of fluid stays intact as it passes over a solid surface. Thus, the pressure at the surface can be calculated assuming the surface were a piston moving into a column of fluid. In this work, piston theory is used to calculate the rigid body forces, moments, and stability derivatives of a hypothetical hypersonic vehicle. Only longitudinal motion is considered in this case and lateral motion will be included in subsequent work.

HSV Model

Figure 1 shows the hypersonic vehicle considered in this work. The vehicle consists of 4 surfaces: an upper surface and three lower surfaces. All pertinent lengths and dimensions are in units of feet and degrees, respectively. The goal is to apply piston theory to determine the pressure distribution on the surfaces of the vehicle, which, in turn, can yield the forces and moments. On the upper surface, the surface is modelled as a piston moving into a column of fluid that has the properties of the freestream. On the lower surface, the surface is modelled as a piston moving into a column of fluid that has the properties of the fluid behind the oblique shock. The pressure on the face of a piston moving into a column of perfect gas is

$$\frac{P}{P_\infty} = \left(1 + \frac{\gamma - 1}{2}\frac{V_n}{a_\infty}\right)^{\frac{2\gamma}{\gamma-1}} \quad (1)$$

where the subscript "∞" refers to the steady flow conditions past the surface, V_n is the velocity of the surface normal to the steady flow, a_∞ is the speed of sound, and P is the pressure. Taking the binomial expansion of Eq. 1 produces

$$\frac{P}{P_\infty} = 1 + \frac{2\gamma}{\gamma - 1}\frac{\gamma - 1}{2}\frac{V_n}{a_\infty} = 1 + \frac{\gamma V_n}{a_\infty} \quad (2)$$

*Electronics Engineer, Control Theory and Optimization Branch, 2210 Eighth Street, Ste 21, Email Michael.Oppenheimer@wpafb.af.mil, Ph. (937) 255-8490, Fax (937) 656-4000, Member AIAA

†Senior Aerospace Engineer, Control Theory and Optimization Branch, 2210 Eighth Street, Ste 21, Email David.Doman@wpafb.af.mil, Ph. (937) 255-8451, Fax (937) 656-4000, Senior Member AIAA

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Multiplying through by P_∞ and using the perfect gas law ($P = \rho RT$) and the definition of the speed of sound ($a^2 = \gamma RT$) yields the basic result from first-order linear piston theory

$$P = P_\infty + \rho_\infty a_\infty V_n \tag{3}$$

The infinitesimal force due to the pressure is

$$d\mathbf{F} = -PdA\mathbf{n} \tag{4}$$

where dA is a surface element and $\mathbf{n}$ is the outward pointing normal.

To compute the forces, moments, and stability derivatives, consider small perturbations, from a steady flight condition at M_∞, in the velocities u, v, and w and the rates p, q, and r. Consider first the upper surface. The velocity of a point on the upper surface due to these perturbations is

$$V_u = (V_\infty \cos(\alpha + 3^\circ) + u)\,\hat{\imath} + (V_\infty \sin(\alpha + 3^\circ) + w)\,\hat{k} + \boldsymbol{\omega} \times \mathbf{r}_u \tag{5}$$

where $\hat{\imath}, \hat{k}$ are unit vectors in the x and z body axes, respectively, $\boldsymbol{\omega}$ is the angular rate vector and α is the angle of attack. For longitudinal motion only, $\boldsymbol{\omega} = q\hat{\jmath}$ where $\hat{\jmath}$ is a unit vector in the y body axes direction. In Eq. 5, $\mathbf{r}_u$ is the position vector of a point on the upper surface given by

$$\mathbf{r}_u = r_{ux}\hat{\imath} + r_{uy}\hat{\jmath} + r_{uz}\hat{k} \tag{6}$$

For the lower surface defined by the points c and d in Figure 1, use the velocity of the flow after the oblique shock to obtain

$$V_{l_{cd}} = (V_2 \cos 6.2^\circ + u)\,\hat{\imath} + (-V_2 \sin 6.2^\circ + w)\,\hat{k} + \boldsymbol{\omega} \times \mathbf{r}_{l_{cd}} \tag{7}$$

while for the surface defined by points g and h

$$V_{l_{gh}} = (V_2 + u)\,\hat{\imath} + (-V_2 + w)\,\hat{k} + \boldsymbol{\omega} \times \mathbf{r}_{l_{gh}} \tag{8}$$

where $\mathbf{r}_{l_{cd}}$ and $\mathbf{r}_{l_{gh}}$ are position vectors of a point on the lower surface given by

$$\mathbf{r}_{l_{cd}} = r_{l_{cd}x}\hat{\imath} + r_{l_{cd}y}\hat{\jmath} + r_{l_{cd}z}\hat{k} \tag{9}$$

$$\mathbf{r}_{l_{gh}} = r_{l_{gh}x}\hat{\imath} + r_{l_{gh}y}\hat{\jmath} + r_{l_{gh}z}\hat{k} \tag{10}$$

An expansion fan occurs on the final lower surface and will be considered shortly. The normal vectors for the upper and lower surfaces are

$$
\begin{aligned}
\mathbf{n}_u &= n_{ux}\hat{\imath} + n_{uy}\hat{\jmath} + n_{uz}\hat{k} \\
\mathbf{n}_{l_{cd}} &= n_{l_{cd}x}\hat{\imath} + n_{l_{cd}y}\hat{\jmath} + n_{l_{cd}z}\hat{k} \\
\mathbf{n}_{l_{gh}} &= n_{l_{gh}x}\hat{\imath} + n_{l_{gh}y}\hat{\jmath} + n_{l_{gh}z}\hat{k}
\end{aligned} \tag{11}
$$

Performing the cross products required by Eqs. 5, 7, and 8 gives

$$
\boldsymbol{\omega} \times \mathbf{r}_u = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & q & 0 \\ r_{ux} & r_{uy} & r_{uz} \end{bmatrix} = q r_{uz}\hat{\imath} - q r_{ux}\hat{k} \tag{12}
$$

$$
\boldsymbol{\omega} \times \mathbf{r}_{l_{cd}} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & q & 0 \\ r_{l_{cd}x} & r_{l_{cd}y} & r_{l_{cd}z} \end{bmatrix} = q r_{l_{cd}z}\hat{\imath} - q r_{l_{cd}x}\hat{k} \tag{13}
$$

$$
\boldsymbol{\omega} \times \mathbf{r}_{l_{gh}} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & q & 0 \\ r_{l_{gh}x} & r_{l_{gh}y} & r_{l_{gh}z} \end{bmatrix} = q r_{l_{gh}z}\hat{\imath} - q r_{l_{gh}x}\hat{k} \tag{14}
$$

To determine the velocity of interest, i.e., the velocity of the surface normal to the steady flow, take the dot product of the velocity with the appropriate normal vector and use the result in Eq. 3 to obtain

$$
\begin{aligned}
P_u &= P_\infty + \rho_\infty a_\infty \left(\mathbf{V}_u \cdot \mathbf{n}_u\right) \\
P_{l_{cd}} &= P_2 + \rho_2 a_2 \left(\mathbf{V}_{l_{cd}} \cdot \mathbf{n}_{l_{cd}}\right) \\
P_{l_{gh}} &= P_2 + \rho_2 a_2 \left(\mathbf{V}_{l_{gh}} \cdot \mathbf{n}_{l_{gh}}\right)
\end{aligned} \tag{15}
$$

Substituting the results of Eq. 15 into Eq. 4 gives

$$
\begin{aligned}
d\mathbf{F}_u &= \left\{-P_\infty - \rho_\infty a_\infty \left(\mathbf{V}_u \cdot \mathbf{n}_u\right)\right\} dA_u \mathbf{n}_u \\
d\mathbf{F}_{l_{cd}} &= \left\{-P_2 - \rho_2 a_2 \left(\mathbf{V}_{l_{cd}} \cdot \mathbf{n}_{l_{cd}}\right)\right\} dA_{l_{cd}} \mathbf{n}_{l_{cd}} \\
d\mathbf{F}_{l_{gh}} &= \left\{-P_2 - \rho_2 a_2 \left(\mathbf{V}_{l_{gh}} \cdot \mathbf{n}_{l_{gh}}\right)\right\} dA_{l_{gh}} \mathbf{n}_{l_{gh}}
\end{aligned} \tag{16}
$$

Using (Eqs. 5, 7, and 8) and the appropriate normal vector (Eq. 11), Eq. 16 becomes

$$
\begin{aligned}
d\mathbf{F}_u &= \left(-P_\infty - \rho_\infty a_\infty \left\{\left(V_\infty \cos\left(3^\circ + \alpha\right) + u + q r_{uz}\right) n_{ux} + \left(V_\infty \sin\left(3^\circ + \alpha\right) + w - q r_{ux}\right) n_{uz}\right\}\right) dA_u \mathbf{n}_u \\
d\mathbf{F}_{l_{cd}} &= \left(-P_2 - \rho_2 a_2 \left\{\left(V_2 \cos 6.2^\circ + u + q r_{l_{cd}z}\right) n_{l_{cd}x} + \left(V_2 \sin 6.2^\circ + w - q r_{l_{cd}x}\right) n_{l_{cd}z}\right\}\right) dA_{l_{cd}} \mathbf{n}_{l_{cd}} \\
d\mathbf{F}_{l_{gh}} &= \left(-P_2 - \rho_2 a_2 \left\{\left(V_2 + u + q r_{l_{gh}z}\right) n_{l_{cd}x} + \left(V_2 + w - q r_{l_{gh}x}\right) n_{l_{gh}z}\right\}\right) dA_{l_{gh}} \mathbf{n}_{l_{gh}}
\end{aligned} \tag{17}
$$

The upper and lower surface elements, $dA_i\mathbf{n}_i$ can be written as

$$\begin{aligned} dA_u\mathbf{n}_u &= \left(n_{ux}\hat{\imath} + n_{uy}\hat{\jmath} + n_{uz}\hat{k}\right) dA_u \\ dA_{l_{cd}}\mathbf{n}_{l_{cd}} &= \left(n_{l_{cd}x}\hat{\imath} + n_{l_{cd}y}\hat{\jmath} + n_{l_{cd}z}\hat{k}\right) dA_{l_{cd}} \\ dA_{l_{gh}}\mathbf{n}_{l_{gh}} &= \left(n_{l_{gh}x}\hat{\imath} + n_{l_{gh}y}\hat{\jmath} + n_{l_{gh}z}\hat{k}\right) dA_{l_{gh}} \end{aligned} \tag{18}$$

In order to evaluate the forces, the required integrations must be performed. In order to perform this task, the position vectors and normal surface vectors must be determined. For the upper surface, the position vector can be computed by evaluating the geometry in Fig. 1 to yield

$$\begin{aligned} &\mathbf{r}_u = x\hat{\imath} + \tan 3^\circ\,(x - 55)\,\hat{k} \\ &-45 \leq x \leq 55 \end{aligned} \tag{19}$$

The upper surface normal vector is

$$\hat{n}_u - \sin 3^\circ\hat{\imath} + \cos 3^\circ\hat{k} \tag{20}$$

For the lower surface, these position vectors are

$$\begin{aligned} &\mathbf{r}_{l_{cd}} = x\hat{\imath} + \{-\tan 6.2^\circ\,(x - 8) + b\}\,\hat{k} \\ &8 \leq x \leq 55 \end{aligned} \tag{21}$$

$$\begin{aligned} &\mathbf{r}_{l_{gh}} = x\hat{\imath} + (b + h_i)\,\hat{k} \\ &-12 \leq x \leq 8 \end{aligned} \tag{22}$$

while the normal vectors become

$$\begin{aligned} \hat{n}_{l_{cd}} &= -\sin 83.8^\circ\hat{\imath} + \cos 83.8^\circ\hat{k} \\ \hat{n}_{l_{gh}} &= \hat{k} \end{aligned} \tag{23}$$

To compute the aerodynamic properties of the vehicle, the quantities in Eq. 17 are integrated over the surface of the vehicle. Using Eqs. 17- 19, force on the upper surface is

$$F_u = \int_{-l}^{0}\int_{-45}^{55} \left(-P_\infty + \rho_\infty a_\infty A\right)\left(-\sin 3^\circ\hat{\imath} + \cos 3^\circ\hat{k}\right) dxdz \tag{24}$$

where

$$A = \{(V_\infty\,(3^\circ + \alpha) + u + qx)\tan 3^\circ\,(x - 55) + (V_\infty \sin{(3^\circ + \alpha)} + w - qx)\cos 3^\circ\} \tag{25}$$

For the lower surfaces, the forces become

$$F_{l_{cd}} = \int_b^0 \int_8^{55} (P_2 + \rho_2 a_2 B) \left(-\sin 83.8° \hat{\imath} + \cos 83.8° \hat{k}\right) dxdz \tag{26}$$

where

$$B = \{V_2 \cos 6.2° + u + q\,(-\tan 6.2°\,(x-8) + b)\,(-\sin 83.8°) + (V_2 \sin 6.2° + w - qx) \cos 83.8°\} \tag{27}$$

and

$$F_{l_{gh}} = \int_{-12}^{8} \{-P_2 + \rho_2 a_2 (w - qx)\} \hat{k} dx \tag{28}$$

Performing the integrations produces

$$\begin{aligned} F_u = {} & 100lP_\infty - 5000\,(V_\infty \cos(3° + \alpha) + u)\, l a_\infty \rho_\infty \tan 3° \\ & +100 a_\infty \rho_\infty (V_\infty \sin(3° + \alpha) + w)\, l \cos 3° \\ & + \frac{175000}{3} a_\infty \rho_\infty lq \tan 3° - 500 a_\infty \rho_\infty lq \tan 3° q \left(-\sin(3° + \alpha)\, \hat{\imath} + \cos(3° + \alpha)\, \hat{k}\right) \end{aligned} \tag{29}$$

$$F_{l_{gh}} = 20\,(-P_2 + a_2 \rho_2 \{w + 2q\})\, \hat{k} \tag{30}$$

$$\begin{aligned} F_{l_{cd}} = {} & 47 P_2 b - 47 a_2 \rho_2 b V_2 \cos 6.2° + 47 a_2 \rho_2 b V_2 \sin 6.2° \cos 83.8° \\ & -47 a_2 \rho_2 b^2 V_2 \sin 6.2° \sin 83.8° + \frac{2209}{2} a_2 \rho_2 b \tan 6.2° V_2 \sin 6.2° \sin 83.8° \\ & + \frac{2961}{2} a_2 \rho_2 bq \cos 83.8° \left(-\sin 83.8° \hat{\imath} + \cos 83.8° \hat{k}\right) \end{aligned} \tag{31}$$

Future Work

At this point, the forces on the vehicle have been determined. These will be used to determine the pitching moment and similar analysis will be used to evaluate the stability derivatives. Also, the flexible body effects on the vehicle will be taken into account.

Conclusions

In this work, piston theory is used to develop a model for the longitudinal dynamics of a hypersonic vehicle. In particular, velocities of flow normal to the surface of the vehicle are used in a first order piston theory framework to determine the pressures on the surfaces of the vehicle. The pressures are then integrated over the body to determine the forces acting on the vehicle. Piston theory is useful here because it allows the inclusion of the unsteady aerodynamic effects, which are not captured using other techniques.

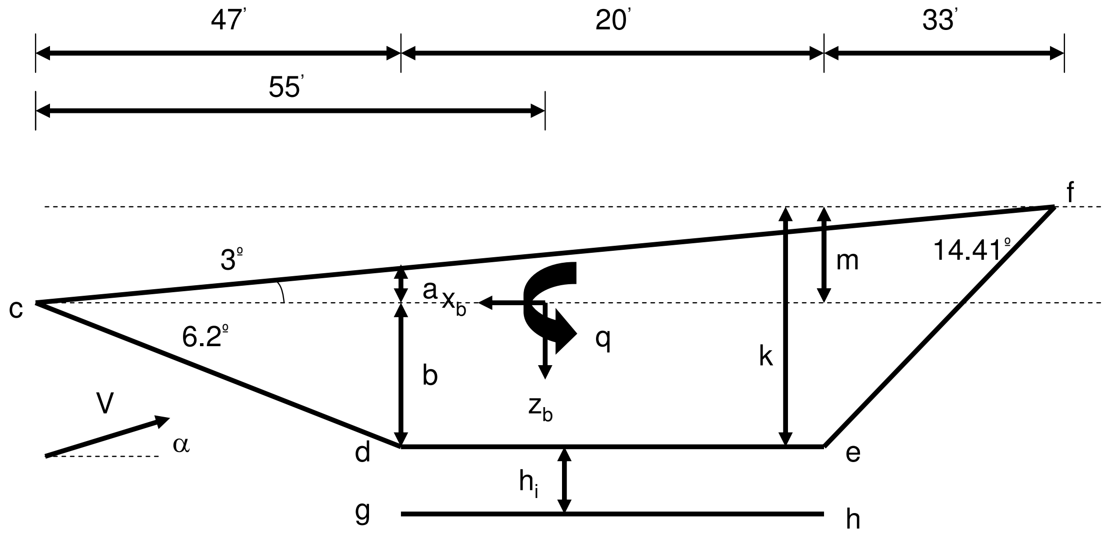

Figure 1: Hypersonic Vehicle.